Limitations of atom-photon squeezing from Bose-Einstein condensates

Thomas Gasenzer, David C. Roberts, and Keith Burnett

Clarendon Laboratory, Department of Physics
University of Oxford
Squeezing I

Generalized optical elements:

- **linear**: mirrors, beam splitters, ...
  \[ a'_j = U^{-1}a_j \ U = \sum_i A_{ji} \ a_i \]

- **non-linear**: squeezers, down-converters, ...
  \[ a'_j = \sum_i (A_{ji} \ a_i + B_{ji} \ a_i^\dagger) + \alpha_j \]

  e.g.
  - single-mode squeezers: \( S = \exp[-\xi \ a^\dagger_1^2 + \text{h.c.}] / 2 \)
  - two-mode squeezers: \( D_2 = \exp[-\zeta \ a_1^\dagger \ a_2^\dagger + \text{h.c.}] \)
Squeezed states

Single-mode squeezing:

\[ S(\xi) \ket{0} = \sqrt{\text{sech} \; \xi} \sum_{n=0} \sqrt{[(2n)!]/n!} \left[ -\frac{1}{2} \tanh \; \xi \right]^n \ket{2n} \]

\[ \Rightarrow \langle x_\lambda \rangle = 0, \quad (\Delta x_{m\pi/2})^2 < (\Delta x_{(m+1)\pi/2})^2, \quad x_\lambda = \frac{\hat{a}e^{-i\lambda} + \hat{a}^\dagger e^{i\lambda}}{\sqrt{2}} \]

Two-mode squeezing:

\[ D_2(\zeta) \ket{0,0} = \text{sech} \; \zeta \sum_{n=0} \left[ -\tanh \; \zeta \right]^n \ket{n,n} \]

\[ \Rightarrow \langle n_1 - n_2 \rangle = 0, \quad (\Delta [n_1-n_2])^2 = 0 \]
Squeezing III

Bloch-Messiah reduction [Braunstein ’99]*

of arbitrary combinations of linear and non-linear elements:

\[
\begin{align*}
&U \\
&\cong \\
&V' \\
&\approx
\end{align*}
\]

*) S.L. Braunstein, quant-ph/9904002

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Squeezing IV

Bloch-Messiah reduction of

\[ D_2 \cong S \]

BS = 50:50 beam splitter
Atom-photon squeezing

Two-mode squeezer: (Bragg transition in BEC)

\[ \text{if } k=0: \quad D_2 = \exp\left(-i\Delta \sqrt{N_0} \left[ \hat{a}_s^\dagger \hat{b}_{\Delta k}^\dagger + \text{h.c.} \right] \right) \]

internally excited state

\[ \Delta \]

scattered photons: \( k_s, \omega_s \)

recoiling atoms: \( k + \Delta k, \omega_k + \Delta \omega \)

\[ \Delta k = k_L - k_s, \quad \Delta \omega = \omega_L - \omega_s \]

atomic c.m. mode \( k, \omega_k \),
e.g. \( k=0 \) (condensate)

\[ \text{Laser, } k_L, \omega_L \]
Weakly interacting BEC

Hartree-Fock-Bogoliubov:

\[ H_{\text{BEC}} = \sum_{k} \omega_{k} \beta_{k}^{\dagger} \beta_{k} + \text{higher order terms} \]

quasiparticle transformation:

\[ \beta_{k} = u_{k} \hat{b}_{k} + v_{k} \hat{b}_{-k}^{\dagger} \]

Bogoliubov quasiparticle frequencies:

\[ \omega_{k} = \mu \sqrt{k^{2} + k_{0}^{2}} \]

\[ k \equiv k/k_{0}, k_{0} \equiv \sqrt{8\pi a n_{0}}, \mu \equiv \frac{\hbar k_{0}^{2}}{2m} \]
A-γ squeezing in HFB I

Atom-photon pair production: densities

\[ \tilde{\Omega} = 1 \text{s}^{-1}, N_0 = 10^6 \]
A-γ squeezing in HFB II

Atom-photon pair production: squeezing

\[ \xi_3 = \left[ \Delta (n_{\Delta k}^b - n_{ks}^a) \right]^2 / \langle n_{\Delta k}^b + n_{ks}^a \rangle, \quad \xi_{1,2} = \left[ \Delta (a_{s}^{\dagger} b_{\Delta k} \pm \text{h.c.}) \right]^2 / \langle n_{\Delta k}^b + n_{ks}^a \rangle \]
Squeezing parameter

The squeezing parameter compares to the classical (uncorrelated) case:

\[
[\Delta(n_1 - n_2)]^2 = [\Delta n_1]^2 + [\Delta n_2]^2 - 2\langle n_1, n_2 \rangle,
\]

\[
\langle n_1, n_2 \rangle = \langle \hat{n}_1 \hat{n}_2 \rangle - \langle \hat{n}_1 \rangle \langle \hat{n}_2 \rangle
\]

In the classical case:

\[
[\Delta n_1]^2 + [\Delta n_2]^2 = \langle \hat{n}_1 \rangle + \langle \hat{n}_2 \rangle,
\]

\[
\langle n_1, n_2 \rangle = 0.
\]
Quasiparticle decay

Effective equations of motion incl. decay:

\[
\frac{\partial}{\partial t} \hat{\beta}_k = -i \left( \omega_k - \frac{i}{2} \gamma_k \right) \hat{\beta}_k + \hat{F}_k
\]

Decay widths at \( T=0 \):

\[
\gamma_k \sim (\mu k_0 a / 4)
\]

\( \propto k^5 \)
A-γ squeezing & decay I

Atom-photon pair production: densities

\[ \tilde{\Omega} = 1 \text{s}^{-1}, N_0 = 10^6 \]

\[ \langle n^b_{\Delta k} \rangle \]

\[ \langle n^a_k \rangle \]

\[ \langle n^b_{-\Delta k} \rangle \]

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A-γ squeezing & decay II

Relative occupation number:

\[ \log_{10} \langle n_A - n_B \rangle \]

\[ \log_{10} (t/\text{sec}) \]
A-γ squeezing & decay III

Atom-photon pair production: squeezing

\[ \xi_3 \equiv \frac{[\Delta(n^b_{\Delta k} - n^a_{k_s})]^2}{\langle n^b_{\Delta k} + n^a_{k_s} \rangle}, \quad \xi_{1,2} \equiv \frac{[\Delta(a^+_s b_{\Delta k} \pm \text{h.c.})]^2}{\langle n^b_{\Delta k} + n^a_{k_s} \rangle} \]
Conclusions

Atom-photon relative number squeezed states may be produced in scattering off BECs. The number of produced particle pairs is greatly enhanced in this case.

Squeezing is limited by
- initial collisional correlations
- rescattering of atoms and photons

Atomic recollisions may be described as quasiparticle damping. This limits the achievable minimum value of the relative number variance.
The Wigner function of the squeezed and entangled two-mode state

\[ D_2(r|0,0\rangle = \text{sech } r \sum_{n=0}^{\infty} [\tanh r]^n |n, n\rangle \]

reads:

\[ W(\alpha_1; \alpha_2) = \frac{4}{\pi^2} \exp\{- e^{-2r} [(x_1 - x_2)^2 + (p_1 + p_2)^2] - e^{2r} [(x_1 + x_2)^2 + (p_1 - p_2)^2]\} \]

\[ \underset{r \rightarrow \infty}{\longrightarrow} C \delta(x_1 + x_2) \delta(p_1 - p_2) \]

\[ \text{continuous variable entanglement} \]
Continuous variable entanglement II

Condition for correlations:

\[ \gamma_{\Delta k} \ll 4\tilde{\Omega}\sqrt{N_0}(u_{\Delta k} - v_{\Delta k}) \]

This is fulfilled in the considered case, where

\[ \gamma_{\Delta k} / 4\tilde{\Omega}\sqrt{N_0}(u_{\Delta k} - v_{\Delta k}) \approx 10^{-1} \]

E.g., for significantly smaller \( \tilde{\Omega} \), or larger \( n_0 \) the continuous variable correlations disappear \( (\gamma_k \propto n_0^{3/2}) \).