Quantum correlations in stimulated molecular dissociation

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Motivation

- add parametric down-conversion of matter waves to the arsenal of nonlinear atom optics

- would give a resource similar to production of correlated photon pairs in $\chi^{(2)}$-nonlinear optics

- quantum effects expected:
  - particle number-difference squeezing (√)
  - quadrature squeezing
  - EPR correlations and Bell inequalities with massive particles
  - applications to precision measurements
Photo-dissociation of a molecular BEC

- start with a BEC of molecular dimers

- coherently dissociate the molecules into atom pairs, using coherent Raman transitions (reverse to Raman photoassociation: [PRL 84, 5029 (2000)]).

\[
\hbar |\Delta| = \frac{\hbar^2 k^2}{2m_1}
\]

Momentum conservation: \( \pm k_0 = \sqrt{2m_1 |\Delta|/\hbar} \)
Schematic diagram

- similar to multimode travelling-wave parametric down-conversion in $\chi^{(2)}$-nonlinear optics
Parametric field theory (1D)

\[ H_0 = \sum_{i} \int dx \left[ \frac{\hbar^2}{2m_i} |\nabla \hat{\Psi}_i|^2 + \hbar \Delta \hat{\Psi}_i^\dagger \hat{\Psi}_i + V_2 \hat{\Psi}_2^\dagger \hat{\Psi}_2 \right] \]

\[ H_{int} = \frac{\hbar \chi}{2} \int dx \left[ \hat{\Psi}_2 \hat{\Psi}_1^\dagger \hat{\Psi}_1^\dagger + H.c. \right] \]

\[ H_{self} = \sum_{i} \frac{\hbar U_{ij}}{2} \int dx \hat{\Psi}_i^\dagger \hat{\Psi}_j^\dagger \hat{\Psi}_j \hat{\Psi}_i \]

- \( \hat{\Psi}_{1/2}(t, x) \) – atomic/molecular field operators
- \( \chi \) – molecule formation rate (\( M = A_2 \rightleftharpoons A + A \))
- \( U_{11} = 4\pi \hbar a_{11}/m_1, \quad a_{11} \) – 1D S-wave scattering length
- \( 2\hbar \Delta = (2E_1 - E_2) - \hbar (\omega_2 - \omega_1) \) – two-photon detuning (like phase mismatch in optics)
\[ (x) \nabla \equiv (x) \nabla u^{z_1} \Omega + \nabla \left( (x) \nabla \Phi (x) \frac{\partial}{\partial \Phi} z_1 \Omega + \nabla \right) \]

\[ (x', \gamma) \chi \equiv (x) \nabla u \wedge (\gamma) \chi \leftarrow (x) \nabla \Phi (\gamma) \chi \]

for the molecular BEC density \( n_2 \).

- Replace \( \frac{\partial}{\partial \Phi} \) by a \( c \)-number; assume a Thomas-Fermi profile.
- Atoms produced limit to short interaction times, and small numbers of atoms produced.
- Neglect the molecular field depletion approximation.

Approximations
Quantum dynamics

- Stochastic $P$-representation equations:

\[
\frac{\partial \psi(x, t)}{\partial t} = i \frac{\hbar}{2m_1} \frac{\partial^2 \psi}{\partial x^2} - i \tilde{\Delta} \psi + \tilde{x} \psi^+ + \sqrt{\tilde{x}} \eta_1
\]

\[
\frac{\partial \psi^+(x, t)}{\partial t} = -i \frac{\hbar}{2m_1} \frac{\partial^2 \psi^+}{\partial x^2} + i \tilde{\Delta} \psi^+ + \tilde{x} \psi + \sqrt{\tilde{x}} \eta_2
\]

\[
\tilde{x} = x(t) \sqrt{n_2(x)}
\]

\[
\tilde{\Delta} = \Delta + U_{12} n_2(x)
\]

\[
< \eta_i(x, t) \eta_j(x', t') > = \delta_{ij} \delta(t-t') \delta(x-x') - \text{noise term}
\]

- solve numerically - can also include depletion
Relative particle number squeezing

- Quantum correlations:

\[
V = \frac{[\Delta(\hat{N}_- - \hat{N}_+)]^2}{\langle \hat{N}_- \rangle + \langle \hat{N}_+ \rangle} = 1 + \left[ \langle (\hat{N}_+)^2 \rangle - \langle \hat{N}_- \hat{N}_+ \rangle \right] / \langle \hat{N}_+ \rangle
\]

- normalized variance of fluctuations in the particle number difference (with \( \Delta \hat{X} \equiv \hat{X} - \langle \hat{X} \rangle \)), where

\[
\hat{N}_{+(-)}(t) = \int_{0(-\infty)}^{+\infty(0)} dx \hat{\Psi}^\dagger_1(x) \hat{\Psi}_1(x)
\]

- Quantum squeezing corresponds to: \( V < 1 \)

- 'Twin' beams = (separation) + (\( V < 1 \))
Example of twin atomic beams

To allow atoms to escape the effective trap \( \Delta(x) = \Delta + U_{12}n_2(x) \), need: \( |\Delta| \gg |U_{12}|n_2(0) \)
Example of trapped beams

Strong attractive ($U_{12} < 0$) atom-molecule $S$-wave scattering: $U_{12} n_2(x)$ acts like a trapping potential.
More realistic example

- $a_{22} = a_{11} = 5.4 \text{ nm}$, $a_{12} = -4.6 \text{ nm}$ ($^{87}\text{Rb}$)

- molecular BEC size: $50 \mu\text{m}$

  aspect ratio: 100

- initial number of molecules: $1.5 \times 10^4$

  number of atoms: 100

- dissociation time: 1.7 ms
SUMMARY
(cond-mat/0110556)

- scheme for producing twin atom-laser beams
- matter-wave analog of parametric down-conversion
- quantum squeezing in relative particle number
- robust against losses, phase diffusion
- destructive measurement of particle number in one beam produces a single beam with a well-defined particle number
- possible applications: EPR correlations, Bell inequalities with massive particles, quadrature squeezing, precision measurements