Measurement of Moment of Inertia

Objective

In this experiment the moments of inertia of various rotating systems are experimentally found by using the relation

\[ \tau_{\text{Net}} = I \alpha \]

The moments of inertia are then calculated by using the appropriate theoretical equations. Then the experimental and theoretical values can be compared.

Theory

For the rotating equipment shown in Figure 1, the total moment of inertia is

\[ I_{\text{TOTAL}} = I_{\text{PLATFORM}} + I_{\text{ROD}} + 2 I_{\text{CYL}}. \]  

where

\[ I_{\text{ROD}} = \frac{1}{12} M_{\text{ROD}} L_{\text{ROD}}^2 \]

and the parallel axis theorem can be used for each cylinder.

This gives

\[ I_{\text{CYL}} = I_{\text{ABOUT ITS C.M.}} + M_{\text{CYL}} X^2 \]

\[ I_{\text{CYL}} = \frac{1}{2} M_{\text{CYL}} R_{\text{CYL}}^2 + M_{\text{CYL}} X^2 \]

Where the length or distance X is shown in Figure 1.

Figure 1. Rotation apparatus with string wrapped around the spool and passing over the smart pulley to the hanging mass. Note the steel pin to catch and hold a loop in the string wound around the spool.

Figure 2. A close up of the rotation apparatus. Each cylinder is bolted to the sixth hole from the center of the rod. “X” is the distance from the axis of rotation to the center of either cylinder. NOTE: In the procedure the cylinders are to be placed in position 3 and 8. The cylinders are currently shown in position 6.
The net Torque, \( \tau_{\text{net}} \), is related to the moment of inertia, \( I \), and the net angular acceleration, \( \alpha \), via the relation

\[
\tau_{\text{net}} = I \alpha. \tag{4}
\]

Applying Newton’s second law to the descending mass \( m \), (as shown in Figure 1)

\[
mg - T = ma
\]

where \( a \) is its acceleration, and \( T \) is the string tension.

So the string tension is

\[
T = m(g - a) \tag{5}
\]

There is some friction in the bearings of the rotation apparatus. This can be measured separately by finding the load (\( m_f \)) needed to make the apparatus rotate at constant speed. Its weight then exactly balances friction, so the friction force is

\[
f = mg,
\]

where \( m_f \) is the “friction mass.”

The net force accelerating the apparatus is then

\[
F_{\text{NET}} = T - f \tag{7}
\]

and the net torque is

\[
\tau_{\text{NET}} = F_{\text{NET}} R_{\text{spool}} \tag{8}
\]

The angular acceleration is

\[
\alpha = \frac{a_{\text{TAN}}}{R_{\text{spool}}} \tag{9}
\]

where \( a_{\text{TAN}} \) is the tangential component of the acceleration of the surface of the spool that the string is wrapped around, which equals the string acceleration (the radial component of the acceleration, the centripetal acceleration, does not enter into the calculation).
Pre-Lab Assignment

1. Study the theory, procedure, and calculations in the lab handout.

2. List the standard units of the key quantities a, T, τ, α, and I.

3. Calculate the total moment of inertia of the system shown in Figure 2 given the following information:

<table>
<thead>
<tr>
<th>Cylinder Data</th>
<th>Rod Data</th>
<th>Rotating Apparatus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass each cylinder = 3.0 kg</td>
<td>Length = 80 cm</td>
<td>Moment of inertia = .005 kg m²</td>
</tr>
<tr>
<td>Diameter each cyl. = 6.0 cm</td>
<td>Mass = 1 kg</td>
<td></td>
</tr>
<tr>
<td>C.M. of each cyl. located at 10cm from axis of rotation</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Overview

First, we will determine the "frictional mass;" this is the weight (mg) required to overcome friction of the rotating apparatus. Then, we will exert torques on the base of the apparatus, and measure the angular acceleration produced by each torque. The graph of torque versus angular acceleration will allow a determination of the total moment of inertia of the apparatus. The total moment of inertia of the apparatus measured experimentally will be compared to that predicted using equation 1. The moment of inertia of the apparatus is the sum of the moment of inertia of each part (see equation 1). The contribution from the rod and the cylinders can be calculated, while that of the platform will be measured separately. The moment of inertia of the apparatus will be determined for two different configurations of the apparatus.

Equipment

Science Workshop 750 Interface
Smart pulley and Pasco Universal Clamp
Moment of Inertia Apparatus
Mass and bar attachment
5 gram mass holder
50 gram mass holder
One full weight set included the following slotted masses:
2 x 200 gm 1 x 100 gm 1 x 50 gm 3 x 20 gm 2 x 10 gm 2 x 5 gm
Multi-filament 50 lb test string
Large Triple-Beam balance (up to 4 kg)
Vernier Caliper

IMPORTANT NOTE: Use \( g = 9.801 \text{ m/s}^2 \) to ensure proper accuracy in your calculations.
Procedure and Analysis for Moment of Inertia Experiment

I. Set-up of computer and interface

1. Start up Data Studio
2. Select "Smart Pulley" as the sensor.
3. Be sure velocity (m/s) is selected and click and drag a graph to the icon.

II. Set-up of equipment

Assemble the apparatus as shown in Figure 2. (Please note that in Fig. 2 the cylinders are in position 6.) Place the cylinders in position 3 as measured from the center of the rod. Clamp the smart pulley to the lab table, and plug its stereo phone plug into channel 1 of the interface. Catch a loop tied in one end of the string on the steel pin protruding from the spool portion of the base. Wind the string around the spool two or more time, then lead it over the smart pulley and attach the 5g hanger to it (do not use tape to attach the string to the spool).

III. Mass and Length measurements

1. Measure the mass and diameter of each of the cylinders to determine the average mass and radius of the cylinders. Use the average cylinder diameter to determine the radius.
2. Measure the mass and length of the rod that holds the cylinders.
3. Measure the length (2X) between the centers of the two upright cylinders for positions 3 and 8.
4. Measure the diameter of the spool with calipers, and determine the radius of the spool. Note, check and double-check this measurement.

IV. Measurement of friction in the rotation apparatus

1. Suspend a 5g hanger on the string. Increase the mass on the hanger in 5g increments until the apparatus starts to rotate. When the apparatus starts to rotate, record the rotation velocity as a function of time on the computer. Apply a linear fit to the data, and obtain the slope of the best fitting line. The acceleration should be so close to zero that we will take it to be zero, and you have just determined the "frictional mass," described earlier. You should do this measurement twice to insure that you have an accurate measure of the "frictional mass." The weight of the "friction mass" is equal to the force needed to overcome the friction experienced by the bearing on which the apparatus rotates. This weight (mg) must be subtracted from each tension to obtain the net force that will cause the platform to rotate (see equation 7).

V. Measuring the Acceleration of the Descending Mass

1. Replace the 5 gram holder with the 50 gram holder.
2. Set the hanging mass equal to 300 grams plus the friction mass.
3. Begin to record data, and release the mass.
4. When the mass gets close to the floor, stop recording data.
5. Click on Scale to Fit to change the scale of the graph and view all the data.
6. Click and drag a rectangle enclosing all the points you want to use to calculate the best straight line fit. Find the Fit Button, and select Linear. The slope of the line is the acceleration of the descending mass.
7. Record the acceleration in an Excel spreadsheet.
8. You should also record any other information you will need into the spreadsheet, such as the mass that caused the acceleration.
9. Print a sample graph to include in your laboratory report.

VI Collecting additional data

1. Repeat the steps outlined in V with a hanging load of 400 grams plus the friction mass.
2. Repeat this for 500 grams and 600 grams, so you have total of 4 data points to plot when you produce the plot of $\tau$ versus $\alpha$.
3. Plot $\tau$ versus $\alpha$, and obtain the experimental value for the moment of inertia $I$ of the apparatus for this configuration.
4. Move the cylinders to position 8, set the hanging mass equal to 400 grams plus the friction mass, and repeat the steps outlined in V. Repeat this using 500, 600, and 700 grams.
5. Calculate $\tau$ and $\alpha$ from your data, and plot $\tau$ versus $\alpha$ to obtain an experimental value for the moment of inertia $I$ of the apparatus for this configuration.
6. To compare these experimental values with predicted values, we will need to determine the moment of inertia of the platform of the apparatus (see equation 1).

VII. Measurement of the Moment of Inertia of the platform

1. Remove the bar and cylinders from the apparatus.
2. Repeat IV to find the mass needed to overcome the friction of the platform base, the "friction mass." This will probably be different from that determined earlier because the load on the base is different.
3. Using the 5 gram holder set the load to the friction mass plus 50 grams.
4. Measure the acceleration.
5. Record this into an Excel spreadsheet, along with other information you will need, such as the hanging mass that caused the acceleration.
6. Repeat with 60, 70, and 80 grams. (If the data does not form a very straight line, use the lower velocity data to find the accelerations.)

Calculations of Moment of Inertia (theoretical values)

1. Use equations 1, 2, and 3 to calculate the total moment of inertia of the apparatus when the cylinders were in position 3. Remember to use the measured moment of inertia of the platform determined in part VII. Compare this with the experimental value and find the percent difference.
2. Repeat this for the cylinders in position 8.

Calculations of Moment of Inertia (experimental data)

See step 5 of Part VI. Be sure to plot a graph for both positions of the cylinders (two graphs).
Report

The predicted and experimentally determined total moment of inertia should be compared in your laboratory report. The percent difference between them should also be discussed for both runs. The report should also include a discussion of the sources of error in this experiment.